LIMITS [1.8] & CONTINUITY [1.7]

NON-TECHNICAL DEFINITION OF A LIMIT (Math 124):

We define the limit of the function \( f(x) \) as \( x \) approaches \( c \), written \( \lim_{x \to c} f(x) \), to be a number \( L \) (if one exists) such that \( f(x) \) is as close to \( L \) as we want whenever \( x \) is sufficiently close to \( c \) (but \( x \neq c \)). If \( L \) exists, we write \( \lim_{x \to c} f(x) = L \).

Example 1: Explain why \( \lim_{x \to 0} \left( \frac{1}{x^2} \right) \) does not exist.

As \( x \) approaches zero, \( \frac{1}{x^2} \) becomes arbitrarily large, so it cannot approach any finite number \( L \).

Therefore we say \( \frac{1}{x^2} \) has no limit as \( x \to 0 \) and we write: \( \lim_{x \to 0} \left( \frac{1}{x^2} \right) \) DNE where \( DNE \equiv \) Does Not Exist.

If, however, \( \lim_{x \to c} f(x) \) does not exist because \( f(x) \) gets arbitrarily large on both sides of \( c \), we also say \( \lim_{x \to c} f(x) = \infty \).

Since \( \frac{1}{x^2} \to \infty \) as \( x \to 0^+ \) and \( \frac{1}{x^2} \to \infty \) as \( x \to 0^- \), we also write \( \lim_{x \to 0} \left( \frac{1}{x^2} \right) = \infty \).

DEFINITION OF CONTINUITY

The function \( f \) is continuous at \( x = c \) if the following principles hold:

1) \( f \) is defined at \( x = c \), that is \( (c, f(c)) \) is a point on the graph of \( f \).

2) \( a \) \( \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) \)

\( [b] \) \( \lim_{x \to c} f(x) = f(c) \)

Example 2: Let \( g(x) = \begin{cases} (x + 1)^2 & \text{if } x \leq 1 \\ x & \text{if } x > 1 \end{cases} \) Is \( g(x) \) continuous at \( x = 1 \)?

Example 3: Let \( h(z) = \frac{5z^2 + 2}{z^2 + 1} \) Is \( h(z) \) continuous at \( z = 3 \)?